Unemployment and education in an overlapping-generations economic growth model

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We introduce labor unions and unemployment into an overlapping-generations model with endogenous fertility and endogenous growth and consider the cost of time spent child-rearing. A tax subsidy for education has no effect on fertility and the existence of a threshold at which the growth rate is maximized owing to unemployment and the funding of education is financed through income tax. The threshold is lower than those in previous studies because this model incorporates unemployment, whereas previous research assumes full employment. The ineffectiveness of education tax against fertility may arise as the cost of time spent child-rearing is a constant.

Keywords: fertility, unemployment, economic growth, overlapping-generations model

Subject classification codes: J13, I25, J64

1. Introduction

Education and economic growth present an interesting combination of topics to macroeconomists. Pioneering studies of education and economic growth include the models proposed by Glomm and Ravikumar (1992, 1997). In developed countries such as Japan, both a low fertility rate and a high unemployment rate are issues of public concern. However, Glomm and Ravikumar's (1992, 1997) model considers the fertility rate and (un)employment rate as exogenous. Zhang and Casagrande (1998) show empirical evidence that a rise in per capita educational subsidies does not increase fertility but does increase the growth rate. As Zhang and Casagrande (1998) point out, subsidizing education reduces its cost and increases the relative cost of child-rearing. However, if the subsidy for education is financed by (flat) income tax, then it would reduce the cost of child-rearing by lowering the opportunity cost of spending time on rearing children. Overall, the two opposing effects cancel out each other. Eventually, the subsidy for education would have no effect on fertility.

Zhang and Casagrande (1998) do not consider unemployment. However, many developed countries experienced high unemployment in the early 21st century. Ahn and Mira (2002) show that unemployment has a stronger income effect than a substitution effect. Relatively high and persistent unemployment is explained, in part, by a combination of high income tax and a strong labor union. Additionally, Daveri and Tabellini (2000) show the correlation between the income tax rate and unemployment rate.[1]

Thus, if the subsidy for education is financed by income tax, then it would be necessary to examine what happens to unemployment, fertility, and economic growth. Likewise, it would be imperative to investigate if higher unemployment rates caused by a high income tax are associated with lower fertility. Furthermore, it is also relevant to study if the economic growth rate is affected by a high unemployment rate.

Fanti and Gori (2010) consider fertility as endogenous and report interesting results. However, their model considers both the economic growth rate and the unemployment rate as exogenous. As mentioned above, macroeconomists tend to believe that education

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[1] In Japan, the ratio of organized labor is only 17 percent, according to a basic survey of labor unions by Japan's Ministry of Health, Labour and Welfare. However, in Japan, *Shunto*, a large labor negotiation, affects the wages of many companies that do not have labor unions. Additionally, Daveri and Tabellini (2000) show that labor unions' wage negotiation hypothesis holds in many developed countries including Japan. Therefore, even in a country in which the ratio of organized labor is low, such as Japan, we can state that labor unions' wage negotiation hypothesis holds at least partly.

is an engine of economic growth. Their model would be more interesting if the growth rate was endogenous, as in Zhang and Casagrande (1998).

Additionally, Fanti and Gori's (2010) model assumes that the unemployment rate is given (zero). However, especially in Japan, a high unemployment rate and a low economic growth rate pose a problem to policymakers. A better result could be obtained if the correlation between the unemployment rate and growth rate is revealed. As mentioned earlier, without unemployment, a subsidy for education financed by income tax would have no effect. However, if unemployment exists, does it still have no effect?

In this study, we consider an endogenous economic growth rate by extending the research of Fanti and Gori (2010) in which the rate of growth is exogenous. They introduce public education in their model. Public education, comprising both primary and secondary education, is an important part of education in many developed countries such as Japan. We assume that education is solely delivered by the public sector. We also consider the unemployment rate as endogenous, following Daveri and Tabellini (2000). We prove the existence of a threshold at which the growth rate is maximized, since investment in education is financed by income tax and unemployment exists. From this perspective, this study extends the model of Glomm and Ravikumar (1997) with endogenous unemployment and birth rates. By introducing physical capital, this study can be considered as an extension of Zhang and Casagrande (1997). Additionally, in our model, a labor union negotiates for wages, thereby indicating the existence of unemployment.

2. Model

2.1 Household

Consider a two-period overlapping-generations model in which a representative household in t generation (t period) has preferences defined by $c_{1,t}$, $c_{2,t+1}$, and n_t, that is, consumption during youth, consumption at old age, and number of children (i.e., birth rate), respectively, following previous studies such as Fanti and Gori (2010).

$$\max_{c_{1,t}, c_{2,t+1}, n_t} (1-\phi)\ln c_{1,t} + \rho \ln c_{2,t+1} + \phi \ln n_t \tag{1}$$

Here, $0 < \phi < 1$ is the parameter of the utility function and $0 < \rho < 1$ is the time preference.

The government collects a tax on labor income and lump-sum taxes, where the tax rate on labor income is $0 < \theta < 1$ and the lump-sum tax rate is τ_t. The youth earn income as follows: a worker in his/her youth is employed with probability l_t. A young worker obtains after-tax income of $(1-\theta)(1-mn)h_t l_t w_t - \tau_t$, where h_t is per capita human capital. However, he/she has periods in which he/she resigns for childcare or avails childcare leave. He/she uses some of the available time (defined as unity) mn_t, where $m > 0$ is the cost of raising a child per young person. If he/she is not employed, he/she collects unemployment benefit $(1-l_t)p_t$ (p_t is the unemployment benefit).

Therefore, his/her income in youth is $(1-\theta)(1-mn_t)h_t w_t l_t - \tau_t + (1-l_t)p_t$. Some of this income is consumed in youth, that is, $c_{1,t}$, and the rest is saved, that is, s_t. In his/her old age, he/she consumes $(1+r_{t+1})s_t$ with interest $r_{t+1}s_t$. Therefore, his/her budget constraints during his/her youth and old age are given by

$$c_t + s_t = (1-\theta)(1-mn_t)h_t w_t l_t - \tau_t + (1-l_t)p_t$$

and

$$c_{2,t+1} = (1+r_{t+1})s_t, \tag{2}$$

respectively. From (2), the lifetime budget constraint is

$$c_t + \frac{1}{1+r_{t+1}}c_{2,t+1} = (1-\theta)(1-mn_t)h_t w_t l_t - \tau_t + (1-l_t)p_t. \tag{3}$$

Maximizing his/her utility subject to (3), savings, s_t, and number of children per household, n_t, are derived as follows:

$$s_t = \frac{\rho}{1+\rho}\left[(1-\theta)\,h_t w_t l_t - \tau_t + (1-l_t)\,p_t\right] \tag{4}$$

and

$$n_t = \frac{\phi}{1+\rho}\frac{1}{m\,(1-\theta)\,h_t l_t w_t}\left((1-\theta)\,h_t l_t w_t + (1-l_t)\,p_t - \tau\right), \tag{5}$$

respectively.

Equation (4) indicates that a representative household's savings, s_t, are a proportion of $(1-\theta)\,h_t w_t l_t - \tau_t + (1-l_t)\,p_t$, which is the income when he/she has no children.

2.2 Firm

A representative firm maximizes its profit

$$\max_{K_t, l_t L_t} AK_t^{\alpha}\left((1-mn_t)\,h_t l_t L_t\right)^{1-\alpha} - (1+r_t)\,K_t - (1-mn_t)\,h_t l_t w_t L_t, \tag{6}$$

in which K_t is the capital stock, h_t is the quality of an employee (which improves with education), l_t is the employment rate, L_t is the population, r_t is the interest rate (we assume that the capital stock decreases in a term), w_t is the wage rate per efficient unit of labor, $0<\alpha<1$ is the capital share (i.e., $1-\alpha$ is the labor share), and A is the scale parameter. By dividing equation (6) by L_t, the equation can be expressed as

$$\max_{k_t, l_t} Ak_t^{\alpha}\left((1 - mn_t) h_t l_t\right)^{1-\alpha} - (1 + r_t) k_t - (1 - mn_t) h_t l_t w_t. \tag{7}$$

Maximizing its profit, the first-order conditions of (7) are

$$1 + r_t = \alpha A k_t^{\alpha-1}\left((1 - mn_t) h_t l_t\right)^{1-\alpha}$$
$$(1 - \alpha) A k_t^{\alpha}\left((1 - mn_t) h_t l_t\right)^{-\alpha} = w_t. \tag{8}$$

Equation (9) is also the firm's labor demand function, which can be solved in terms of l_t, that is,

$$l_t = \left(\frac{(1 - \alpha) A}{w_t}\right)^{\frac{1}{\alpha}} \frac{k_t}{(1 - mn_t) h_t}. \tag{9}$$

Differentiating equation (9) with regard to labor income tax rate θ , we can derive

$$\frac{dl_t}{dw_t} = -\frac{1}{\alpha}\frac{l_t}{w_t}. \tag{10}$$

2.3 Labor union

We describe the behavior of the labor union. For simplicity, all individuals belong to the labor union. We assume that this labor union is sufficiently strong to determine wages, but not to determine lump-sum tax τ_t, unemployment benefit p_t, labor income tax θ , or interest rate r_t.[2] The labor union maximizes a union member's expected

[2] We assume that the labor union has a 'myopic view' because it does not consider the fact that the labor income tax and lump-sum tax are affected by wages.

income $(1-\theta)(1-mn_t)h_t w_t l_t - \tau + (1-l_t)p_t$ subject to (10). Taking interest rate r_t as given, this is equivalent to maximizing labor union members' indirect utility $v((1-\theta)(1-mn_t)h_t w_t l_t - \tau + (1-l_t)p_t)$. Solving the union's maximization problem, we derive the following equation:

$$h_t w_t = \frac{p_t}{(1-\theta)(1-\alpha)(1-mn_t)}. \tag{11}$$

Equation (11) equates a union member's wage times his/her human capital with the product of unemployment benefit p_t and some mark-up ratio $1/[(1-\theta)(1-\alpha)(1-mn_t)]$. Unemployment benefit p_t is assumed to be a proportion of product per capita, that is, $\delta A k_t^{\alpha}((1-mn_t)h_t l_t)^{1-\alpha}$. From this and equation (8), employment rate l_t is derived:

$$l_t = \frac{(1-\theta)(1-\alpha)^2}{\delta}. \tag{12}$$

2.4 Government

2.4.1. Unemployment benefit. Unemployment benefit p_t is financed by lump-sum tax τ_t. A proportion of product per capita $\delta A k_t^{\alpha}((1-mn)h_t l_t)^{1-\alpha}$ (where τ_t is a variable and δ is a constant) is paid to unemployed workers. That is,

$$L_t \tau_t = L_t(1-l_t)p_t = L_t(1-l_t)\delta A k_t^{\alpha}((1-mn)h_t l_t)^{1-\alpha}. \tag{13}$$

From equations (4) and (13),

$$s_t = \frac{\rho}{1+\rho}[(1-\theta)h_t w_t l_t] \tag{14}$$

and

$$n = \frac{\phi}{1+\rho}\frac{1}{m}, \tag{15}$$

which is constant.

2.4.2. Education. The government provides education $g_t > 0$ to children financed by the labor income tax. For simplicity, we describe the government's budget constraint per young person. Then,

$$g_t = \theta(1 - mn) h_t l_t w_t. \tag{16}$$

3. Equilibrium: results

3.1 Human and physical capital

Human capital per capita at time $t+1$, h_{t+1}, depends on time t, h_t, as well as the education the government provides, g_t:

$$h_{t+1} = Bh_t^{\mu} g_t^{1-\mu} = Bh_t^{\mu}(\theta(1 - mn) h_t l_t w_t)^{1-\mu}. \tag{17}$$

Here, B is a scale parameter of education and $0 < \mu < 1$ is a technical parameter of education.

In the equilibrium, the capital stock satisfies $K_{t+1} = S_t \equiv s_t L_t$. Divided by L_t, that is, the population at time t, the capital stock becomes $n_t k_{t+1} = s_t$. Furthermore, from equations (5) and (14), the capital stock becomes

$$k_{t+1} = \frac{\rho m}{\phi}(1-\theta)\,h_t l_t w_t, \tag{18}$$

which does not converge in one period, as seen in Fanti and Gori (2010). Therefore, this economy has transition dynamics, as in Glomm and Ravikumar (1997).

Following Glomm and Ravikumar (1997), and from equations (17) and (18), h/k , which is a constant in the equilibrium, is

$$\frac{h_{t+1}}{k_{t+1}} = \frac{Bh_t^{\mu}\left(\theta(1-mn)\,h_t l_t w_t\right)^{1-\mu}}{\frac{\rho m}{\phi}(1-\theta)\,h_t l_t w}$$
$$= Q(1-\theta)^{-1}\,\theta^{1-\mu}(1-\theta)^{-\mu}(1-\theta)^{\alpha u}\left(\frac{h_t}{k_t}\right)^{\alpha\mu}, \tag{19}$$

where $Q \equiv \frac{B\phi}{\rho m}(1-mn)^{1-\mu(1-\alpha)}(1-\alpha)^{-\mu}A^{-\mu}\left(\frac{(1-\alpha)^2}{\delta}\right)^{-\mu(1-\alpha)}$ is a constant unaffected by labor income tax rate θ . From the fact that $0<\alpha<1$ and $0<\mu<1$, $\frac{h}{k}$ converges at a steady state:

$$\frac{h}{k} = Q^{\frac{1}{1-\alpha\mu}}(1-\theta)^{-\frac{1}{1-\alpha\mu}}\,\theta^{\frac{1-\mu}{1-\alpha\mu}}(1-\theta)^{-\frac{\mu}{1-\alpha\mu}}(1-\theta)^{\frac{\alpha\mu}{1-\alpha\mu}}$$
$$= Q^{\frac{1}{1-\alpha\mu}}\theta^{\frac{1-\mu}{1-\alpha\mu}}(1-\theta)^{\frac{-(1+\mu(1-\alpha))}{1-\alpha\mu}}. \tag{20}$$

It is necessary to interpret equation (20). If labor income tax θ increases, ratio x increases because of a decrease in physical capital k . The second term implies that the increase in θ increases government spending g , human capital h , and ratio x . The third term implies that the increase in θ decreases employment ratio l and physical capital k and then increases x . The fourth term implies that the increase in θ spurs the labor union to hike wage w and savings s , while the physical capital stock k increases and x relatively decreases. The first and second terms are affected by the change in employment ratio l, whereas the third and fourth terms are not affected by the change in l. This classification is used in Section 4. Moreover, birth rate n is a constant and labor income tax rate θ has no impact on it.

3.2 Employment rate

By solving equation (12), we derive

$$l_t = \frac{(1-\theta)(1-\alpha)^2}{\delta}. \tag{21}$$

Differentiating equation (21) with regard to labor income tax rate θ yields

$$\frac{dl_t}{d\theta} = \frac{-(1-\alpha)^2}{\delta} < 0.$$

Owing to the increase in the labor income tax, labor unions demand higher wages, which decrease a firm's willingness to hire and thereby increase the unemployment rate.

3.3 Effect on fertility

We can easily investigate the effect of education (or income tax for education).

Proposition 1. Public education does not affect the fertility rate.

Proof. This is evident from equation .

Zhang and Casagrande (1997) prove that income tax for education has two opposite effects. First, it reduces the cost of education and increases the relative cost of raising a child. Then, it reduces the number of children. This is an inverse relationship between the number of children and quality of children.[3] Second, income tax for education reduces the opportunity cost of spending time on rearing children; therefore, it increases the number of children (another effect overlooked by previous studies). Third, it reduces the rate of employment. It also reduces the opportunity cost of raising children, which increases the number of children. The negative effect for fertility and the positive effects cancel out completely. Therefore, income tax for education has no effect.

3.4 Effect on the growth rate

In this section, we extend the study of Fanti and Gori (2010) to not only the (un)employment rate but also the growth rate. This enables us to analyze the effects of an increase in the labor income tax for education. From equations (8), (12), (18), and (20), the growth rate is calculated as

$$g = \ln k_{t+1} - \ln k_t = S + (1 - \alpha) \ln \theta + (1 - \alpha) \ln x \,, \tag{22}$$

where

$S \equiv \ln \dfrac{\rho m}{\phi} (1 - \alpha) A (1 - mn)^{-\alpha} + \dfrac{(1 - \alpha)}{1 - \alpha\mu} lnQ$ is a constant unaffected by labor income tax rate θ.

Differentiating equation (22) with regard to labor income tax rate θ, we derive the following propositions.

Proposition 2. When labor income tax rate θ is sufficiently low, that is, $\theta < \bar{\theta}$, an increase in labor income tax rate θ raises the growth rate.

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[3] In this study, the representative household does not choose between the quality and quantity of children. However, as human capital rises, the cost of child-rearing also rises and hence the number of children reduces. We cannot say that this is a choice between the quality and quantity of children; however, there is an inverse relationship between the quality and quantity of children.

Proposition 3. $\bar{\theta}$ is always lower than that shown in Glomm and Ravikumar (1997), in which the threshold rate is the labor distribution rate of the production function.

Proof. Differentiating equation (22) with regard to labor income tax rate θ ,

$\frac{dg}{d\theta} > 0$ if

$(1-\alpha)(1-\mu)(1-\theta)-\theta[(2-\alpha)(1-\alpha\mu)-(1-\alpha)(1+\mu(1-\alpha))] > 0.$

From this,

if $\theta < \frac{1-\alpha}{2-\alpha} \equiv \bar{\theta} < 1, \frac{dg}{d\theta} > 0$. In this study, from equation (6), the labor distribution rate is $1-\alpha$. Considering $0 < \alpha < 1$, it is obvious that $\bar{\theta} < 1-\alpha$. To investigate the propositions, from equation (18), we can also express the growth rate as

$$g \equiv \ln k_{t+1} - \ln k_t = \ln \frac{\rho m}{\phi} + \ln(1-\theta) + \ln x + \ln l + \ln w$$
$$= \ln \frac{\rho m}{\phi} + \ln(1-\alpha)A(1-mn)^{-\alpha} + \ln(1-\theta) + \ln x + \ln l - \alpha \ln x - \alpha \ln l . \tag{23}$$

Differentiating equation with respect to θ yields

$$\frac{dg}{d\theta} = -\frac{1}{1-\theta} + (1-\alpha)\frac{d\ln x}{d\theta} + (1-\alpha)\frac{d\ln l}{d\theta} \tag{24}$$

In equation (24), (1) the first term on the right-hand side implies the income effect of labor income tax θ , which is negative. (2) The second term implies the human capital–physical capital ratio effect, which is positive, as mentioned above. (3) The third term implies the employment ratio effect, which is negative.

Why does the fertility change have no effect in line with Zhang (1997)? Owing to the increase in Proposition 1, the increase in income tax has no effect on fertility. Therefore, there is no effect through fertility.

Overall, effects (1) and (3) are negative and effects (2) are positive. If labor income tax θ is sufficiently low, the positive effect dominates. If θ is too high, the negative effect dominates, that is, the growth rate falls. For this reason, threshold $\bar{\theta}$ exists.

We need to interpret Proposition 3. If it were not for the unemployment effect, the growth-maximizing threshold would be similar to that of Glomm and Ravikumar (1997). Therefore, the effect caused by the change in employment ratio θ decreases the growth rate and threshold of θ. For this reason, threshold $\bar{\theta}$ is smaller than that in Glomm and Ravikumar (1997).

From Propositions 1, 2, and 3, we can conclude that public education is important for economic growth, but spending excess budget on public education is not good for economic growth. Glomm and Ravikumar (1997) propose this conclusion, but in this study, unemployment also decreases the growth rate and growth-maximizing tax rate.

4. Simulation

We can validate the above conclusion using a simple simulation. We set the deep parameters as follows: $\phi = 0.9$, $\rho = 0.1$, $m = 0.1$, $\alpha = 0.33$, $\delta = 0.45$, $\mu = 0.75$, and $B = 2$. Then, $\bar{\theta} = 0.401$. We set income tax rate $\theta = 0.05, 0.1, \cdots, 0.9, 0.95$ and use equations (12), (15), and (22) for the simulation.

From Figure 1, we can see $\theta < \bar{\theta}$ and easily confirm that Proposition 2 is true.

From Figure 2, we can see that employment rate l is a decreasing function of income tax rate θ.

From Figure 3, we can see that growth rate g is an inverse U-shaped function of θ. If $\theta < \bar{\theta}$, g is an increasing function of θ. When $\theta = \bar{\theta} = 0.401$, g is at its maximum and therefore $g = 0.077$. By contrast, if $\theta > \bar{\theta}$, g decreases as θ increases. We can also confirm Proposition 2. It is obvious that the rate is lower than that of Glomm and Ravikumar (1997), where the threshold is $0.67 = 1 - \alpha$. We can also see that Proposition

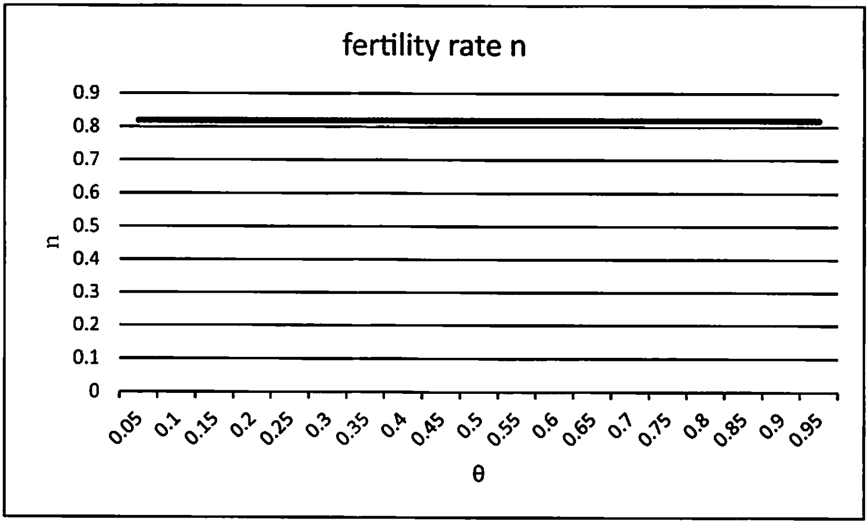

Figure 1. Fertility

Figure 2. Employment rate

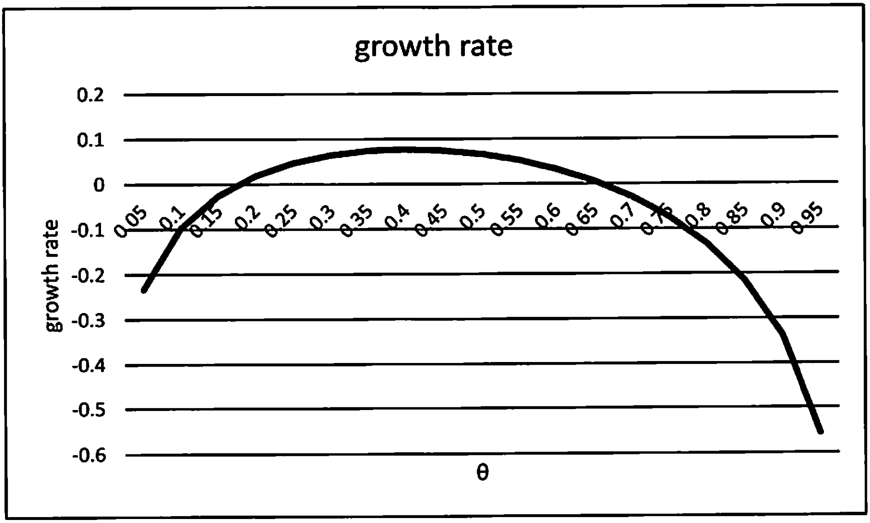

Figure 3. Growth rate

3 holds.

5. Discussion and conclusion

Growth and fertility rates are often examined in a conventional manner, as in previous studies of education and economic growth. However, in developed countries such as Japan, a high unemployment rate and a low fertility rate are issues of public concern. Evidencing this point, this study shows that a growth-maximizing income tax rate exists for labor and that this tax rate is always lower than that shown by Glomm and Ravikumar (1997) owing to the impact of unemployment. It seems interesting that unemployment has no effect on fertility.

We can conclude that unemployment does not have a direct effect on fertility. However, it is harmful to economic growth. Previous studies do not consider the negative effects of unemployment, which may impact on the findings of these studies. This context gives rise to the question of why unemployment has no effect on fertility, contrary to the findings presented by Ahn and Mira (2002). We suppose that the child-rearing cost per young person m is a constant. If we suppose that m is an increasing function of employment ratio l, then fertility n would be a decreasing function of employment.[4]

As mentioned above, we assume a log-linear utility function and that the child-rearing cost is a constant. While these assumptions are essential for simplicity, in the real-world economy, a log-linear utility function and constant child-rearing cost may not hold. A CES-type utility function may thus be suitable. While it is not easy to solve this kind of model analytically, we may be able to carry out a numerical simulation.

While this study assumes that labor unions cause unemployment, other causes are also posited, as in the efficient wage hypothesis and search hypothesis. However, it is not possible to articulate which hypothesis explains real unemployment, and further research is required to examine these hypotheses. If the other hypothesis holds, then the optimal policy may change. For example, if the search-matching hypothesis holds, the optimal policy may be the fluidization of the labor market. This essential research will form a part of our future work.

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4 We can consider this condition if m is a procyclical variable; in other words, if the economy is in a depression (boom), then m decreases (increases).

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